Parameter Error Propagation in BRDF Derived by Fitting Multiple Angular Observations at Single Sun Position

X. Li (1,2), F. Gao (2,3), J. Wang (1), A. Strahler (2), W. Lucht (4), C. Schaaf (2)

(1) Research Center of Remote Sensing and GIS, Beijing Normal Univ., Beijing 100875, China
Tel. 86-10-62532395, Fax: 86-10-62206955, e-mail lixw/wangjd@public.bta.net.cn
(2) Center for Remote Sensing and Dept Geography, Boston Univ., Boston, MA 02215, USA,
Tel. +1-617-353-9678, Fax +1-617-353-3200, e-mail lix/fgao/alan@crsa.bu.edu
(3) Nanjing Institute of Geography and Limnology, CAS, Nanjing 210008, China
(4) Potsdam Institute for Climate Impact Research, POB 601203, D-14412 Potsdam, Germany.

Abstract – In most of the recently developed linear kernel-driven BRDF (bidirectional reflectance) models, there are usually 3 unknowns for each band. Usually a least square (LS) approach is employed for inversion. Assuming the observations are well sampled over the viewing hemisphere (viewing zenith angle θ_v, azimuthal difference ϕ with the solar zenith angle θ_i) for a single θ_i, as in most cases of space-borne multiangulare observations such as POLDER and MISR, the LS solution can be obtained for the three unknowns.

It was once hoped that if the kernel-driven model has sound physical meaning, the three parameters estimated from such good 2-D sampling can be used over the whole 3-D $(\theta_i, \theta_v, \phi)$ bidirection space (3DBS for short).

However, inversion of 395 BRDF datasets acquired by POLDER of CNES (France) shows that when we apply the inversion results over the whole 3-D space, for example, at a far different θ_i, the estimation errors in parameters will propagate differently and thus yield different pattern of prediction errors, independent of the soundness of the BRDF model physics. Our analysis concludes that general knowledge of BRDF shapes of the land surface has to be applied to constrain the inversion of single (or narrow-range) θ_i multiangular observations. [1]

INTRODUCTION

With advance of multiangle remote sensing, it's more and more likely that BRDF models can and will be inverted for important biological or climatological parameters earth surface such as leaf area index and albedo [1]. In order to do so, linear kernel-driven BRDF models were designed for BRDF and albedo products of satellite (real or pseudo) multiangle-viewing instruments, as briefly reviewed in [2]. A linear kernel-driven BRDF model usually has the following form:

$$BRDF = f_{iso} + f_{vol} * k_{vol}(\theta_i, \theta_v, \phi) + f_{geo} * k_{geo}(\theta_i, \theta_v, \phi) \quad (1)$$

where k_* are "kernels", i.e., known functions of θ_i. θ_v, and ϕ; f_* are three unknown coefficients to be adjusted to fit observations. To conserve space, we will write this equation as $Y[M] = K[M,3]X[3]$ where $Y[M]$ is BRDF vector for M observation geometries, $K[M,3]$ is corresponding kernel matrix, and $X[3]$ the parameter vector. Because we concentrate here on error propagation, we'll assume the above model is perfect in physics, i.e., no modeling errors, and therefore both X and Y are true values of parameters and BRDF. M observation geometries can be regarded as M points in 3DBS. In case of single-θ_i observations, these M points distribute only on a 2-D subspace (single-θ_i 2DsS for short). Owing to the nonlinearity of k_{vol} and k_{geo} as functions over this 2DsS, a good sampling over this 2DsS can obtain well-invertible matrix K'K, then a regression method can get the estimations of the unknown X:

$$X_{est} = [K'K]^{-1}K'Y_{obs}, \quad (2)$$

where subscript *obs* mean real observations which inevitably contain noise, usually assumed white and additive. We'll denote the noise vector $\eta[M]$ and its variance σ^2. Then the parameter error vector $X_{err} = [e_{iso}, e_{vol}, e_{geo}]'$ will be:

$$X_{err} = X_{est} - X = [K'K]^{-1}K'\eta \quad (3)$$

Using the estimated parameter vector, then a prediction of BRDF at any point of the 3DBS can be made:

$$y_{pre} = K_{pre}[1,3] * X_{est}[3] \quad (4)$$

Actually any linear combination $A[1,n] * Y[n]$ over any subspace of the 3DBS, such as blacksky albedo (BSA) or white sky albedo (WSA), can be written in similar form:

$$u_{pre} = A * Y_{pre} = AK_{pre}X_{est} = U_{pre}[1,3] * X_{est}[3]. \quad (5)$$

$$u_{err} = U_{iso}e_{iso} + U_{vol}e_{vol} + U_{geo}e_{geo} \quad (6)$$

Note this is a single random number, its variance σ_u^2 consists of variances and covarinces of three parameter errors weighted by corresponding U terms. Lucht and Lewis [2] applied the concept of weight of determination as a powerful tool to study the propagation of noise to error variance of any u_{pre}. Based on their works, we further analyze the effect of parameter estimate errors on relation of two predictions, to better our understanding on how the noise in

[1] This work was funded under China's Key-importance Basic Research Program (95-Y-38), CNSF-49971059, 863-308-14-03 (8), and USA's NASA projects NAS5-31369, NAG5-7217.

0-7803-6359-0/00/$10.00 © 2000 IEEE

the single-θ_i sampling may propagate through the 3DBS, and influence the shape of retrieved BRDF and BSA.

We have to look into this effect because single-day MISR and POLDER obtain multiangular views at almost the same θ_i, and even multi-date POLDER BRDF database or mutidate MODIS/MISR combinations have significant percentages of θ_i ranging smaller than 10°. Note however, this is not such a problem when the illumination geometry is also well sampled such as with MODIS AM&PM measurements.

PROPAGATION OF PARAMETER ERRORS

Following [2], we define a local prediction error $u_{1.err}$ of u_{pre} over a point or a subspace of the same sampling 2DsS. For example, if $y_{1.pre} = K_1[1,3] * X_{est}[3]$ is the prediction of BRDF at a point in the sampling 2DsS, its local predication error $y_{1.err}$ can be obtained by its kernels $K_1[1,3]$ by (6). Now if we want to predict BRDF of another point y_2 which has the same viewing direction (θ_v, ϕ), but a slightly different $\theta_2 = \theta_i + \Delta\theta_i$. we can obtain $y_{2.pre} = K_2[1,3] * X_{est}[3]$ similarly and have:

$$y_{2.pre} - y_{1.pre} = (K_2 - K_1) * X_{est} = \Delta_{real} + \Delta_{err} \quad (7)$$

where Δ_{real} is the true value of $y_2 - y_1$, and

$$\Delta_{err} = (k_{2.geo} - k_{1.geo})e_{geo} + (k_{2.vol} - k_{1.vol})e_{vol} \quad (8)$$

Note that e_{iso} is missing since it's the common part of two predictions

Here we are concerned with Δ_{real} rather than the absolute value of y_2, because we are not only concerned with the variance of y_2, but also the shape of retrievd BRDF. In the case where Δ_{err} is larger than Δ_{real} and with a reversed sign, the real changing tendency of BRDF with θ_i will be reversed in the retrieved BRDF. On the other hand, correctly predicting the increment Δ_{real} is all why we need a BRDF which is assumed to be superior to a Lambert one. At this point, if a lambertian model is applied, it's predication error will be Δ_{real}.

Δ_{real} is linear combination of the true parameters:

$$\Delta_{real} = f_{geo}\Delta k_{geo} + f_{vol}\Delta k_{vol} \quad (9)$$

where $\Delta k_{geo} = k_{2.geo} - k_{1.geo}$ in (8), Δk_{vol} is defined similarly. The eqs (8) and (9) look very similar, but by assumption of perfect model physics, both f_{geo} and f_{vol} should be positive. By requirement of orthogonality, Δk_{geo} and Δk_{vol} should have opposite sign - otherwise the model can not fit different changing tendency. In other words,

$$|\Delta_{real}| = ||\Delta k_{geo} f_{geo}| - |\Delta k_{vol} f_{vol}|| \quad (10)$$

On the other hand, e_{geo} and e_{vol} may be positively or negatively correlated, depending on the LS fitting. Even in the case where perfect 2DsS sampling implies their independence:

$$\sigma_\Delta^2 = (e_{geo}\Delta k_{geo})^2 + (e_{vol}\Delta k_{vol})^2 \quad (11)$$

This propagation error variance can be much larger than Δ_{real}^2 in mathematics and in practice using POLDER BRDF data [3]. For comparison, if Δk_{geo} and Δk_{vol} have the same sign, $|\Delta_{real}|$ will be sum of two positive terms, thus less relative estimation error is guaranteed. However, the expense of this will be loss of ability to fit opposite changing trends with θ_i – or it has to allow either f_{geo} or f_{vol} being negative.

In short, through sampling and data fitting, the noise uncertainty propagates into three parameter errors and their covariances, which further propagate by their own kernels. For single θ_i sampling, this distribution is determined merely by the LS fitting on 2DsS of variable ϕ and θ_v, thus the effects of this uncertainty-sharing on the propagation along θ_i, i.e., through Δk_{geo} and Δk_{vol}, is not constrained. In such sampling scheme, if the model is near-orthogonal to θ_i change, the error propagation can yield large predication σ_Δ^2, and thus a poor retrieval of BRDF shape. The essence of the problem is the lack of information from 2DsS sampling about change of θ_i, rather than a problem of (near) orthogonality of kernels. This error in Δy, or in average, in the derivative of BSA w.r.t. θ_i (BSA'), may yield uncertainty such as the whole BRDF shape up and down at other solar zenith angles. In general, when taking y_1 as a random point on the sampling 2DsS and y_2 as also a random point over the 3DBS, we may assume independence of y_1 and Δy, then:

$$y_{2.err} = y_{1.err} + \Delta_{err} \quad (12)$$

and:

$$\sigma_2^2 = \sigma_1^2 + \sigma_\Delta^2 \quad (13)$$

In other words, given the total information from the observations available, the total uncertainty of predictions is fixed. Different sampling schemes can minimize either local prediction uncertainty or propagation uncertainty, but their sum can not be minimized unless you increase the total information. Based on this version of "Principle of Uncertainty", it's obvious that a single-θ_i sampling provides more information on the sampling 2DsS of y_1, leaving a larger share of uncertainty to propagation.

CONSTRAINED SINGLE-θ_i INVERSION

Therefore the key problem of single-θ_i inversion is the lack of information from observations about the change tendency with θ_i. So this is a situation we called "well-conditioned (in single-θ_i 2DsS) but still singular (in dimension of θ_i)."

For real multiangle satellite instruments such as MISR or POLDER, the θ_i is almost fixed. For quasi-multiangle (multi-dates) POLDER BRDF database [3], there are 56

(14% of 395) data sets with θ_i range smaller than 5°, for example. By such narrow range of θ_i, a moderate noise level (say RMSE 0.01) may result in a failed inversion.

In such cases, *a priori* knowledge on BRDF of land surface should be injected into inversion. Our knowledge based "single-look BRDF inversion" algorithm [4] should be applicable in a "single-θ_i inversion" as well. However, single-θ_i cases usually have enough looks for a well-conditioned inversion over the 2DsS and thus should need much smaller "*a priori* information ratio" (a-R) rather than 3/4 suggested for single-look inversion. We analyzed well sampled Parabola field measurements (Deering and Leone, cited in [4]) and found there is a good relationship between the BRDF shapes derived from single-θ_i sampling and that from the full data set. Generally speaking, single-θ_i 2DsS sampling usually yields a sharper bowlshape or domeshape BRDF than full 3DBS data set. This knowledge can then be used to constrain single-θ_i inversion. A cost function $(BSA'_e - BSA'_b)^2/\sigma_b^2$ and a corresponding lateral look can be made [4], where the subscript e means estimated from the unknown parameters and b means from the *a priori* best guess. This single θ_i algorithm results in an a-R of 1/NOL (number of looks). In our tests of the algorithm using Parabola data, the improvement of single θ_i inversion is marvelous even when the ratio is as small as 1/100. The following presents some typical examples:

θ_i	nol	f_{iso}	f_{vol}	f_{geo}	rmse	bsa	wsa
18.0	115	.137	.238	.013	.014	.124	.166

This presents a rather strong bowlshape in the sampling 2DsS, thus it predicts a much higher WSA based on BSA observed at near nadir θ_i. After one lateral look is added, the results are changed to

θ_i	nol	f_{iso}	f_{vol}	f_{geo}	rmse	bsa	wsa
18.0	116	.170	.120	.049	.017	.126	.134

We can see the changes in RMSE (0.003) and BSA (0.002) is minor, but the changes for the three parameters are significantly improved and more reliable. Another one before adding the lateral look is:

θ_i	nol	f_{iso}	f_{vol}	f_{geo}	rmse	bsa	wsa
73.1	90	.317	.027	.132	.024	.119	.163

This presents a domeshape at large θ_i, thus a prediction overestimated WSA and f_{iso} (which is the nadir sun and nadir viewing reflectance). After the lateral look is added, the results are now:

θ_i	nol	f_{iso}	f_{vol}	f_{geo}	rmse	bsa	wsa
73.1	91	.173	.065	.057	.031	.115	.116

Note the changes in RMSE (0.007) and BSA (0.004) are larger than the previous one - the reasons may be two: our current *a priori* means favour a bowlshape [4], thus constraining wild domeshape would cost more; and the NOL is smaller than before. But please note that the domeshape is more or less still reserved.

However in case where NOL is even smaller, a lateral look may inject more *a priori* information than needed. For example,

θ_i	nol	f_{iso}	f_{vol}	f_{geo}	rmse	bsa	wsa
31.7	51	.520	-.12	.228	.029	.286	.223

This presents a strong domeshape. After the lateral look is added:

θ_i	nol	f_{iso}	f_{vol}	f_{geo}	rmse	bsa	wsa
31.7	52	.341	.160	.071	.049	.276	.286

We can note though we get more reliable WSA and f_{iso}, the increment of RMSE is rather large, and the strong domeshape has been reversed into a weak bowlshape. All this indicates the ratio 1/51 may have over constrained the inversion.

CONCLUSION AND DISCUSSION

Single sun position sampling has intrinsic lack of information about BSA change tendency with θ_i. But from Parabola BRDF sets, we have gained some information which can be used to constrain single θ_i inversion. It seems a very small a-R like 1/100 is appropriate for well sampled single θ_i Parabola data. However single date MISR and POLDER have only around 10 looks which may not have enough information to overdetermine the three unknowns in the 2DsS. In such case what a-R would be the most appropriate needs further practice.

ACKNOWLEDGEMENT

The authors thank Drs. D. Deering of NASA (USA), M. Leroy and P. Bicheron of CNES (France) for the datasets, Dr. W. Lucht and Mr. LIU Qiang for email discussions.

REFERENCES

[1] Strahler, A.H., X. Li, S. Liang, J.-P. Muller, M.J. Barnsley, and P. Lewis, MODIS BRDF/albedo product: Algorithm technical basis document, NASA EOS-MODIS Doc., v2.1, 55pp, 1994.

[2] Lucht, W. and P. Lewis, Theoretical noise sensitivity of BRDF and albedo retrieval from the EOS-MODIS and MISR sensors with respect to angular sampling, Int. J. Remote Sensing, 21(1): 81-98, 2000.

[3] P. Bicheron and M. Leroy, BRDF signatures of major biomes observed from space, submitted to J. of Geo. Res., 1999.

[4] Li, X., F. Gao, J. Wang and A. Strahler, *A Priori* Knowledge Accumulation and Its Application to Linear BRDF Model Inversion, Presented in the 2nd IWMMM, Ispra, Italy, September 1999, and submitted to J.G.R.